GRE Math Quick Review - Percentages

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1 Introduction

Percentages complete a trio (also featuring fractions and decimals) of mathematical objects that are both used to represent division or parts in some fashion, and are universally found in daily life. Percentages come to us in the form of interest rates, sports statistics, and clothing store sales. The word percent means just what it looks like it should mean; per hundred, or hundredth (-*cent* being the hundred part). As such, percentages represent hundredths of numbers, and are denoted by the symbol: %.

In reality, a percentage is just a really specific fraction, so much of our work with percentages will actually be work with fractions, which is covered elsewhere. The big challenge with percentage questions is interpreting and transcribing them correctly, as a lot of information can fly around between just a few symbols here. We'll also need to interpret percent increases and decreases, and cover a few useful facts. Onward.

When working percentage questions, we almost certainly begin by converting the percentage to a fraction. This prevents any foulups in interpretation (such as forgetting 17% of something does not mean 17 *times* that something. Now, percent means hundredth, so to convert say, $x\%$ to a fraction, all we have to do is put x in the numerator and 100 in the denominator:

$$x\% = \frac{x}{100} \tag{1}$$

This holds true regardless of what x is, and you may need to reduce or further translate things, as shown in a few examples here:

$$\frac{1}{5}\% = \frac{\frac{1}{5}}{100} = \frac{1}{500} \tag{2}$$

$$0.3\% = \frac{0.3}{100} = \frac{3}{1000} \tag{3}$$

The key, after knowing this technique, is learning how to unravel a statement which involves a percentage. Let's consider the following question and decompose it:

- What is 75% of 500?

Here, we have an unknown, let's call it x. As is typical in mathematics, we verbalize the equals sign (=) with the verb "is." So far, this gives us:

$$x = 75\% \; of \; 500 \tag{4}$$

How about the other word here, "of?" We want 75 percent *of* 500, which really means the fraction $\frac{75}{100}$ *times* 500. This yields:

$$x = \frac{75}{100} \times 500 \tag{5}$$

From here, it is basic algebra/arithmetic to see that $x = 375$.

Almost all percentage questions are stated in this (or very similar ways), the only difference being what piece the unknown is. Two more percentage statements translated:

- 80 is what percent of 200?

Reading left to right, we have 80, =, x% of 200, and writing the percentage as a fraction as done above gives us:

$$80 = \frac{x}{100} \times 200 \tag{6}$$

Again, simple algebra gives us that $x = 40$.

- 32 is 20% of what number?

This time we have:

$$32 = \frac{20}{100} \times x \tag{7}$$

Another manipulation yields $x = 160$.

Finally, we have to ensure our clarity in understanding the concepts of percent increase and decrease. Let's set out some terminology:

- The *initial amount* is whatever quantity we begin with. An example would be: Jan's salary was $200,000 before a 12% raise. Here, the initial amount is $200,000.

- The *percent increase* (or decrease) is the percentage a quantity is raised (or lowered). In the above example, the percent increase would be 12%
- The *numerical increase* (or decrease) is the *actual* amount a quantity is raised (or lowered). For example: The price of the shirt was \$40 before a \$5 rebate. Here, the numerical decrease is \$5.

Given a problem involving percentage increase or decrease, all one has to do is plug in the various parts to the following equations. The key is correctly identifying each piece of the puzzle.

Percent Increase of a Quantity:

$$\frac{numeric\ increase}{initial\ amount} \times 100\% \tag{8}$$

Percent Decrease of a Quantity:

$$\frac{numeric\ decrease}{initial\ amount} \times 100\% \tag{9}$$

For example, if asked to find out the amount of Jan's raise in the example, we would have to identify the amount of the raise as the numerical increase, $\$200,000$ as the initial amount, and 12% as the percent increase. From there, it's a plug and chug:

$$\frac{x}{\$200,000} \times 100\% = 12\% \tag{10}$$

$$\frac{x}{\$200,000} = \frac{12}{100} \tag{11}$$

And algebra gives us that $x = \$24000$.

2 Essential Formulas

As advertised, we begin with the formula for converting a percentage to a fraction (this should be memorized, no questions).

Converting Percentages to Fractions:

$$x\% = \frac{x}{100} \tag{12}$$

The only other required formulas are those for percentage increase and decrease, also discussed above:

Percent Increase of a Quantity:

$$\frac{numeric\ increase}{initial\ amount} \times 100\% \tag{13}$$

Percent Decrease of a Quantity:

$$\frac{\textit{numeric decrease}}{\textit{initial amount}} \times 100\% \tag{14}$$

3 Facts of Use

Most of these facts, while useful, are rarely (if ever) *necessary* to solving problems with percentages, though they can provide helpful intuition and methods for process of elimination techniques. The prominent exception is the first fact:

- For a third time, $x\% = \frac{x}{100}$.
- $x\%$ of $y = y\%$ of x. This can be seen easily if we *convert to fractions*:

$$x\% \ of \ y = \frac{x}{100}y = \frac{xy}{100} \tag{15}$$

$$y\% \ of \ x = \frac{y}{100}x = \frac{xy}{100} \tag{16}$$

- If x is less than y, then the percentage increase needed to raise x to y is larger than the percentage decrease required to lower y to x.
- Decreasing an amount by $x\%$, then by $y\%$ always causes a smaller decrease than reducing once by $(x + y)\%$.
- Increasing an amount by $x\%$, then by $y\%$ always causes a larger increase than raising once by $(x + y)\%$.
- For any number x, $x\%$ of 100 is simply x.

4 Specific Techniques

When working problems containing percentages, the following techniques are often useful:

- It bears repeating once more: in almost all cases, begin by converting percentages to fractions.
- If a problem involves percentages and does not specify a particular number (such as; the value of the stock increased by 30%), plug in 100. Since percentages involve a fraction with a denominator of 100, this will simplify calculations greatly. Do not apply this technique in comparison questions if specific numbers are given.

5 Exercises

1. A bowling ball was initially sold at Store A and Store B for the same price. Store A applied a 10% discount. Store B then applied a 22% discount. Store A responded with an second 10% discount.

Column A: The price of the ball at Store A.
Column B: The price of the ball at Store B.

(A) The quantity in column A is greater. (B) The quantity in column B is greater. (C) The two quantities are equal. (D) The relationship cannot be determined from the information given.

2.
Column A: The price of a microwave, offered on sale at 31$ off.
Column B: The price of the same microwave, offered on sale at $31 off.

(A) The quantity in column A is greater. (B) The quantity in column B is greater. (C) The two quantities are equal. (D) The relationship cannot be determined from the information given.

3. If 40% of x is 720, what is 120% of x?

(A) 288 (B) 900 (C) 1440 (D) 2160 (E) 2800

4. Wallace scored an 85% on his history exam. If there were 120 questions, how many did Wallace answer incorrectly? (Assume each question is of equal weight).

(A) 15 (B) 16 (C) 18 (D) 24 (E) 96

5. Arthur gives 20% of his money to Bethany. Bethany gives 30% of that money to Carlos.
Column A: 25% of Arthur's remaining money.
Column B: 300% of Carlos's share of the money.

(A) The quantity in column A is greater. (B) The quantity in column B is greater. (C) The two quantities are equal. (D) The relationship cannot be determined from the information given.

6. A $\$20,000$ car is on sale at 15% off. What is the total cost after a 5% tax, applied to the sale price?

(A) $\$17,000$ (B) $\$17,850$ (C) $\$18,000$ (D) $\$18,450$ (E) $\$21,000$

6 Solutions and Explanations

1. We're asked to compare the price of a bowling ball at two stores. At Store A, the price has been discounted by 10%, and then again by 10%. At Store B, the price has been discounted just once, by 22%. Two approaches. By one of the key facts above, we know that a discount of 10% followed by one of 10% is less than a discount of $(10+10)\% = 20\%$. This means it is also smaller than a discount of 22%, since 22 is greater than 20. Since Store A has decreased the price less, the price is higher.

Alternatively, we can choose a price for the bowling ball. The most convenient one is $100. The first discount at Store A is then for $10, bringing the price down to $90. The second discount cuts off another $9, leaving a final price of $81. At Store B, the 22% discount lowers the price by $22, to $78. In either case, the price at Store A is greater, so the answer is **A**.

2. In this case, we have to be careful about plugging an arbitrary cost of $100. One number might not be enough to determine the correct answer in this comparison question. Instead, let's think about each column. Column A subtracts 31 *percent* of the initial amount. This change is dependent upon the initial price. On the other hand, Column B subtracts $31 from the initial amount, no matter what that amount is. This prompts us to check some numbers. Two easy prices for the microwave are $100 and $31. If the microwave costs $100, then a discount of 31% is a discount of $31, so that the two discounts are equal (and thus the two prices are equal). We can see why it would be dangerous to only check the value $100 here. Now, if the price is $31, a $31 dollar discount brings the price down to zero, which a 31% discount certainly doesn't do. Thus, the prices are not equal. These two examples show that relationship cannot be determined from the information given, answer **D**.

3. Given that 40% of x is 720, we wish to calculate 120% of x. We could

calculate x directly from the first statement. To do this, we write:

$$\frac{40}{100} \times x = 720 \tag{17}$$

Solving this equation for x yields $x = 1800$. We then write another equation to determine 120% of 1800:

$$\frac{120}{100} \times 1800 = y \tag{18}$$

Performing the calculation gives $y = 2160$.

A simpler method is to see that 120% of x is the same as $3 \times 40\%$ of x. Since we know that 40% of $x = 720$, we simply multiply by 3 to get 2160, answer **D**.

4. Given that Wallace scored an 85% on a 120 question exam, we wish to determine how many questions he got incorrect. Well, if Wallace got 85% correct, he must have gotten $100\% - 85\% = 15\%$ incorrect. Thus, we need to compute 15% of 120:

$$\frac{15}{100} \times 120 = x \tag{19}$$

Performing the calculation gives $x = 18$, answer **C**.

5. To compare these amounts of money, utilize the power of the number 100. Assume Arthur begins with \$100. He gives 20% to Bethany. That's \$20. She gives 30% of that to Carlos, which is:

$$\frac{30}{100} \times \$20 = \$6 \tag{20}$$

Thus, Arthur has $\$100 - \$20 = \$80$ left, and Carlos has \$6. We can then simply compute the values of each column. Column A is 25% of Arthur's remaining money:

$$\frac{25}{100} \times \$80 = \$20 \tag{21}$$

Column B is 300% of Carlos's money:

$$\frac{300}{100} \times \$6 = \$18 \tag{22}$$

Thus, Column A is larger, so the answer is **A**.

6. Let's apply the percent decrease and increase in turn. A 15% decrease on $\$20,000$ is:

$$\frac{15}{100} \times \$20,000 = \$3,000 \tag{23}$$

Thus, the sale price is $\$20,000 - \$3,000 = \$17,000$. We then apply a 5% increase for the tax:

$$\frac{5}{100} \times \$17,000 = \$850 \tag{24}$$

Therefore, the total cost is $\$17,000 + \$850 = \$17,850$, answer **B**.