GRE Math Quick Review - Decimals

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1 Introduction

Decimals are another ubiquitous piece of mathematics, something we're used to seeing in prices at the grocery store, at the gas station, even at sporting events. Fortunately, the ways that decimals are used in everyday life are pretty much the same as in GRE-style mathematics.

The first thing to consider is; what exactly are decimals? You might recognize the prefix "deci-" as meaning ten, and this is a reference to our base-10 number system. When we write down integers, we have the ones place, the tens place, the hundreds place, and so on. Decimals are the natural extension of this system to encompass numbers less than one, or non-integer numbers. Let's go over some notation with an example. Consider the following number:

$$39.74502 \tag{1}$$

Working from left to right, we have a 3 in the tens place and a 9 in the ones place, both familiar friends. The period between 9 and 7 is called the **decimal point**, it signifies that the integer portion of our number is done. The place to the immediate left of the decimal point is always the ones place, and the place to the immediate right is the tenths place, which happens to be occupied by a 7. Finishing up to the right, we have a 4 in the hundredths place, a 5 in the thousandths place, a 0 in the ten-thousandths place, and a 2 in the hundred-thousandths place. We would typically read this number as "thirty-nine point seven four five zero two."

We can essentially treat decimals the same way we treat other numbers, we'll see that performing standard operations is not much more difficult than with integers. The biggest deal is keeping track of where the decimal point is supposed to be, which we'll see how to do shortly. One important fact, useful for bookkeeping, is that zeroes at the end of a decimal number do not change the value of the number. For example:

$$41.33 = 41.330 = 41.330000000000000000000000 \tag{2}$$

This makes sense, because all the zeroes are doing is telling us that there are no thousandths, ten-thousandths, and so on, which was implied by the blank space that was there before. More familiarly, it's obvious that there are no millions in the number 5, so that clearly $0000005 = 5$.

2 Essential Formulas

There really aren't formulas to deal with when working decimals, we're just going to learn a few modifications to our standard procedure for adding, subtracting, multiplying and dividing decimals, so that we can keep track of the decimal point.

Addition and Subtraction of Decimals: Adding a subtracting decimal numbers pretty much as easy as adding or subtracting integers. We take one additional step. Before doing the addition, we append placeholder zeroes at the end of the numbers so that each number has the same number of digits to the right of the decimal point. Once this is done, we can just line up the numbers so that the decimal points are in the same place, add or subtract as usual, and slide the decimal point down. Let's look at two examples:

$$24.3 + 51 + 1.88 = ? \tag{3}$$

Let's add a zeroes on the ends and line the numbers up:

$$\begin{aligned} &24.30 && (4)\\ &51.00 && (5)\\ +\ &1.88 && (6)\end{aligned}$$

Using our powers of addition, this yields 77.18. Note how the decimal point continues to line up:

$$\begin{aligned} &24.30 && (7)\\ &51.00 && (8)\\ +\ &\ 1.88 && (9)\\ =\ &77.18 && (10)\end{aligned}$$

How about a subtraction:

$$10 - 9.134 = ? \tag{11}$$

Looks like we'll need to drop some zeroes on the end of that ten (we'll have to put a decimal point there as well):

$$\begin{aligned} &10.000 && (12)\\ -\ &\ 9.134 && (13)\end{aligned}$$

Then, using standard subtraction (with carrying and such), we get:

$$\begin{aligned} & 10.000 && (14) \\ - \ & 9.134 && (15) \\ = \ & 0.866 && (16) \end{aligned}$$

Multiplication of Decimals: Multiplying decimals isn't any more difficult than adding or subtracting them, we don't even have to add extra zeroes this time. We can multiply decimals with a few easy steps:

1. Count the total number of digits to the right of the decimal points in the numbers being multiplied.
2. Multiply the numbers, as if there were no decimal points.
3. Insert a decimal point into the result so that to the right of the decimal point we have the same number of digits as you counted above

Example time. Suppose we need to multiply 1.45×2.2. First step, count the number of digits to the right of decimal points. I see 3. Now, we perform the multiplication, not worrying about decimal points:

$$\begin{aligned} & 1.45 && (17) \\ \times\ & 2.2 && (18) \\ =\ & 3190 && (19) \end{aligned}$$

We then reinsert the decimal point so that three numbers appear to the right: 3.190. Note that we can't drop the zero until we've inserted the decimal point.

Division of Decimals: I suppose that dividing decimals is technically the trickiest of our operations, but again, it's just a few simple steps to add to our normal method of division:

1. If there are digits to the right of the decimal point in the divisor, move the decimal point to the right until you're left with an integer.
2. Move the decimal point in the dividend an equal number of places
3. Perform the division as if there were no decimal points.
4. If the dividend has a decimal point, move it up, so that the answer has the decimal point in the same position.

Again, it's much simpler to see by example. Suppose we want to divide $36 \div 0.24$. First, we see that there are 2 digits to the right of the decimal point in the divisor.

Thus, we shift the decimal point two places to the right in *both* numbers. This gives us the following division problem:

$$24|\overline{3600} \tag{20}$$

This we complete using standard long division:

$$150 \tag{21}$$
$$24|\overline{3600} \tag{22}$$

Then, since the dividend has no decimal point, the answer is simply 150.

Let's do an example in which the dividend ends up with a decimal point. Suppose we want to divide $31.25 \div 2.5$. We start by moving the decimal point one place to the right in each number, since the divisor has one digit to the right of its decimal point. This yields:

$$25|\overline{312.5} \tag{23}$$

Ignoring the decimal point, we complete the division:

$$125 \tag{24}$$
$$25|\overline{312.5} \tag{25}$$

Now, since the dividend has a decimal point, we slide it up to attain the final answer:

$$12.5 \tag{26}$$
$$25|\overline{312.5} \tag{27}$$

We're almost done. We've technically covered all the operations we need, but it will pay off to look at a special case: **Multiplying and Dividing by Powers of Ten** The great benefit of the decimal system is that since it's part of a base-10 number system, multiplying and dividing by powers of ten is really straightforward.

Obviously, multiplying or dividing by 1 does nothing, it leaves the number alone. When multiplying by powers of ten greater than 1, such as 1000, count the number of zeroes. To perform the multiplication, simply move the decimal point that many places to the right. To divide by the same number, just move the decimal point that many places to left. For example:

$$1234.5678 \times 1000 = 1234567.8 \tag{28}$$

$$1234.5678 \div 1000 = 1.2345678 \tag{29}$$

Now, when multiplying by power of ten less than 1, such as .01, count the number of zeroes left of the decimal point, and add 1. Alternatively, write the power of ten as 0.01 and count the number of zeroes straight up. Then we just reverse things; to multiply, move the decimal point that many places to the left; to divide, move the decimal point that many places to the right. For example:

$$9876.5432 \times .01 = 98.765432 \tag{30}$$

$$9876.5432 \div .01 = 987654.32 \tag{31}$$

These methods will often save a lot of time, and are often tested explicitly on the GRE.

3 Facts of Use

- Zeroes at the end of a decimal number (after the decimal point and all non-zero digits) are meaningless, but can serve as useful placeholders.
- A decimal number with no integer part, such as .424 can be written with a leading zero, as 0.424.

4 Specific Techniques

When working problems containing decimals, the following techniques are often useful:

- Write placeholder zeroes when performing addition or subtraction so that it's easy to line up the decimal points of each number.
- When multiplying or dividing by powers of ten, simply move the decimal point rather than work the entire multiplication or division.
- When comparing decimals, write placeholder zeroes on the left and right so that the decimal points line up. Then, beginning with the leftmost place, compare the corresponding digits of each number. If one is greater, that number is automatically larger. If the digits are equal, move to the next place, working left to right until one number's digit is larger. Consider the following examples:

Suppose we want to compare 12.473 and 9.89. We need to add a placeholder zero to the tens place and the thousandths place of 9.89, yielding 09.890. Then line up the numbers and read right to left:

$$12.473 \tag{32}$$

$$09.890 \tag{33}$$

Well, the first digit (tens place) is enough! 1 is bigger than 0, so $12.473 > 9.89$. Let's try another, comparing .023902 and .023899. Here, we don't need any placeholders, so let's just line 'em up!

$$.023902 \tag{34}$$
$$.023899 \tag{35}$$

OK. Tenths place: both 0. Hundredths place: both 2. Thousandths place: both 3. Ten-thousandths place: ALERT! $9 > 8$. We stop right here and know that $.023902 > .023899$.

5 Exercises

1. For what value of y does the following equation hold?

$$(1.25)(.533) = \frac{(12.5)(.0533)}{y} \tag{36}$$

(A) .01 (B) .1 (C) 1 (D) 10 (E) 100

2. Column A: .327 Column B: .3197

(A) The quantity in column A is greater. (B) The quantity in column B is greater. (C) The two quantities are equal. (D) The relationship cannot be determined from the information given.

3. Column A: $1.2 + 2.8 \times 0.9$ Column B: $(1.2 + 2.8) \times 0.9$

(A) The quantity in column A is greater. (B) The quantity in column B is greater. (C) The two quantities are equal. (D) The relationship cannot be determined from the information given.

4. Compute $7.77 \times 10 \times .01 \times 10000$.

(A) 77.7 (B) 777 (C) 7770 (D) 7777 (E) 77770

5. Compute $42 \div 1.4$.

(A) 2.8 (B) 3 (C) 28 (D) 30 (E) 32

6 Solutions and Explanations

1. We want to determine the value of y for which the following equation holds:

$$(1.25)(.533) = \frac{(12.5)(.0533)}{y} \tag{37}$$

Now, we could perform the multiplication on each side, but this could be quite gory. Notice that the same digits appear on both sides, we've just shifted the decimal place in each case. Working on the right side, we see that 1.25 has been shifted to 12.5, a shift of one place to the right. Similarly, .533 has been shifted to .0533, a shift of one place to the left. Thus, the net shift, after we multiply, will be zero! Thus, y must equal 1, answer **C**.

2. We wish to compare the two decimals, .327 and .3197. We append an extra zero to the first number so that the numbers have the same number of digits. We then have .3270 and .3197. We can then compare each digit, moving left to right. The tenths places match, both 3's. The hundredths places, however, differ. Since $2 > 1$, we know that $.3270 > .3197$. Thus, the quantity in column A is greater, so the answer is **A**.

3. We wish to compare two quantities: $1.2 + 2.8 \times 0.9$, and $(1.2 + 2.8) \times 0.9$. The best approach here is simply straight computation. For column A, we have to multiply first by order of operations:

$$1.2 + 2.8 \times 0.9 = 1.2 + 2.52 \tag{38}$$
$$= 3.72 \tag{39}$$

For column B we add first because of the parentheses:

$$(1.2 + 2.8) \times 0.9 = 4 \times 0.9 \tag{40}$$
$$= 3.6 \tag{41}$$

Then, since $3.72 > 3.6$ (we can compare just as in question 2), the quantity in column A is greater, so the answer is **A**.

4. We wish to compute $7.77 \times 10 \times .01 \times 10000$. Now, since the latter three numbers are powers of ten, they're just going to shift the position of the decimal

point in 7.77. Multiplying by 10 shifts us right by 1 place, multiplying by .01 shifts us left by 2 places, and finally multiplying by 10000 shifts us right by 4 places. This is a total shift of 3 places to the right. Applying that to 7.77 yields 7770, answer **C**. Note that 7777 is incorrect, because it appends an additional seven, rather than a zero.

5. We wish to compute $42 \div 1.4$. We can divide $42 \div 14$ rather easily, this is just 3. Shifting the decimal place to the left yields the final result of 30, answer **D**.