GRE Math Quick Review - Conversions

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1 Introduction

Fractions, decimals, and percentages. They are all ways to represent division, or parts of a whole, and we have different techniques for dealing with each of them. Now, if the people writing the GRE were nice, they would never set a problem that involved two or more of these mathematical objects. Of course, such is not the case. Often on the GRE, we'll need to interpret problems which have some combination of fractions, decimals, and percentages (oh my...). As such, we need to learn how to translate, or convert between these three forms. This way, we can take such "combination" problems and rewrite them as fraction problems, decimal problems, or percentage problems, which we have the tools to solve. There isn't a lot more to say here, we'll just go over the six possible conversions in turn and look at some examples.

Converting Fractions to Decimals:

This is likely the most difficult (in terms of calculation) conversion we'll deal with, so let's get it out of the way first. Recall that a fraction, such as $\frac{13}{20}$ is just another way to write the division problem $13 \div 20$. Well, if we actually carry out the long division, we'll get a decimal:

$$0.65 \tag{1}$$

$$20|\overline{13} \tag{2}$$

Simple process, but the most calculation intensive.

Converting Fractions to Percentages:

Two methods here. The first method is to convert the fraction to a decimal, and then convert the decimal to a percentage (we'll cover how this works soon). This method is very, very straightforward, but it does require two separate conversions.

To convert fractions directly to percentages, remember that percentages are just fractions with denominators of 100. We just need to figure out what the numerator is! Say we want to convert $\frac{21}{5}$ into a percentage. We want to find x such that:

$$\frac{21}{5} = \frac{x}{100} \tag{3}$$

Solving this equation for x, we get $x = 420$, which means $\frac{21}{5} = 420\%$.

Converting Decimals to Fractions:

Recall, decimals are just an extension of "places" in our number system, giving us the tenths place, hundredths place, and so on. Take a decimal like .254, which extends to the thousandths place. It's not hard to see that this number is 254 thousandths. Sounds like a fraction to me: $\frac{254}{1000}$. Thus, the procedure is simple; find out the last place in your decimal to determine the denominator. Then just place your number (without decimal point) in the numerator. Reduce and simplify if you desire. For example, let's convert 5.1688 to fraction form. The last place here is the ten-thousandths place, so our denominator will be 10000, and the numerator we read off as 51688. Our fraction: $\frac{51688}{10000}$.

Converting Decimals to Percentages:

This one is nice and simple. Multiply by 100 and slap on a percent sign. Why is this right? Remember, $x\%$ means $\frac{x}{100}$, right? Now, if we have a number, like 3.2 and what to know what percentage this is, we need to solve for x in the following equation:

$$\frac{x}{100} = 3.2 \tag{4}$$

In other words, we need to know how many hundredths (or "percents") 3.2 is. How do we do that? Multiply by 100. This gives, of course, 320%.

Converting Percentages to Fractions:

Hopefully, this one has been driven into your head completely (if not, read the Percentages review sheet again!). Converting a percentage to a fraction is almost a definition:

$$x\% = \frac{x}{100} \tag{5}$$

For example, $29\% = \frac{29}{100}$. Simple as that.

Converting Percentages to Decimals:

Now, if we multiply by 100 and add a percentage sign to convert from decimals to percentages, how should we go the other way? Both processes are reversible, we divide by 100 and drop the percentage sign. For example:

$$72\% = 0.72 \tag{6}$$

2 Essential Formulas

There aren't any magic formulas today, just the methods above. Below, we give a table of useful values to know, written as fractions, decimals, and percentages.

3 Facts of Use

The numbers and conversions in the following table are common enough that they warrant being memorized (many of them may already be familiar to you).

Fraction	Decimal	Percentage
0/1	0	0%
1/20	0.05	5%
1/10	0.1	10%
1/8	0.125	12.5%
1/6	0.1666...	16.666...%
1/5	0.2	20%
1/4	0.25	25%
3/10	0.3	30%
1/3	0.333...	33.333...%
3/8	0.375	37.5%
2/5	0.4	40%
1/2	0.5	50%
3/5	0.6	60%
5/8	0.625	62.5%
2/3	0.666...	66.666...%
7/10	0.7	70%
3/4	0.75	75%
4/5	0.8	80%
5/6	0.8333...	83.333...%
7/8	0.875	87.5%
9/10	0.9	90%
1/1	1	100%
3/2	1.5	150%
2/1	2	200%

4 Specific Techniques

When working problems involving conversions, the following techniques are often useful:

- When dealing with more than two numbers, make the fewest conversions possible. For example, if you have three numbers expressed as percentages and one as a fraction, convert the fraction, rather than the three percentages.
- The most difficult/dangerous conversion is from fraction to decimal, since it involves long division. In this case, you may consider converting all decimals to fractions, since this is usually simpler.
- Before converting a fraction to a decimal (or to a percentage), reduce the fraction to make the long division simpler.
- Play to the strengths of each form. Fractions are easy to multiply, decimals are easy to add, and percentages are great with round numbers (like 100).

5 Exercises

1. Compute the following expression:

$$\frac{3}{4} + 0.15 + 1.32 + 1.53 \tag{7}$$

(A) 3.75 (B) 3.81 (C) 3.83 (D) 3.88 (E) 5.22

2. c is a positive number.

Column A: $\frac{3}{7} \times c$
Column B: 41% of c

(A) The quantity in column A is greater. (B) The quantity in column B is greater. (C) The two quantities are equal. (D) The relationship cannot be determined from the information given.

3. A particular television costs \$1000 at retail price. Salesman Horace offers me a 12% discount. Saleswoman Gretchen offers me the television for $\frac{7}{8}$ of the retail price. How much money is saved by taking Horace's offer?

(A) \$0 (B) \$5 (C) \$10 (D) \$25 (E) \$50

4.

Column A: $\frac{1}{20} + \frac{1}{5} + \frac{1}{2}$
Column B: $0.06 + 0.22 + 0.53$

(A) The quantity in column A is greater. (B) The quantity in column B is greater. (C) The two quantities are equal. (D) The relationship cannot be determined from the information given.

5. Timothy, Ursula, and Victoria each have 100 marbles. Timothy gives you $\frac{1}{50}$ of his marbles. Ursula and Victoria both give you 8% of their marbles. How many marbles did you receive in total?

(A) 10 (B) 18 (C) 24 (D) 36 (E) 56

6. x and y are positive numbers.

Column A: 19% of x
Column B: $\frac{0.5}{2.43} \times y$

(A) The quantity in column A is greater. (B) The quantity in column B is greater. (C) The two quantities are equal. (D) The relationship cannot be determined from the information given.

6 Solutions and Explanations

1. We wish to add a fraction and three decimals. It makes sense to convert the lone fraction to a decimal, since this would be just a single conversion, and since decimals are easy to add. $\frac{3}{4}$ is a fairly common fraction, one you should probably memorize/have memorized. If not, you can do simple long division:

$$0.75 \tag{8}$$
$$4|\overline{3} \tag{9}$$

We then add the decimals as usual:

$$0.75 \tag{10}$$
$$0.15 \tag{11}$$
$$1.32 \tag{12}$$
$$+\ 1.53 \tag{13}$$
$$\overline{3.75} \tag{14}$$

And we see that the answer is **A**.

2. Since c is a positive number, and appears in both columns, we'll definitely be able to determine which column is greater. Let's convert the fraction $\frac{3}{7}$ to a percentage so that we can compare:

$$\frac{3}{7} = \frac{x}{100} \tag{15}$$

Solving for x, we get a value a bit larger than 42 (it is exactly $42\frac{6}{7}$). Well, that's definitely bigger than 41, so the quantity in column A is larger, answer **A**.

3. The problem tells us that Horace's offer is better, one less thing to worry about. Let's compute the price offered by each salesperson. For Horace, we have a 12% discount. This represents a percent decrease: the numeric decrease is:

$$\frac{12}{100} \times \$1000 = \$120 \tag{16}$$

Thus, Horace's price is $\$1000 - \$120 = \$880$.

Gretchen's price is $\frac{7}{8}$ of the retail price:

$$\frac{7}{8} \times \$1000 = \$875 \tag{17}$$

Subtracting the two prices, we get $\$880 - \$875 = \$5$, or answer **B**. Note that we never actually had to make a conversion here! Don't do more work than you have to! Making the conversion would have been more helpful if we were comparing the two prices.

4. We wish to compare the sum of three fractions and the sum of three decimals. The easy way to approach this problem is to notice that the first fraction is less than the first decimal, the second fraction is less than the second decimal, and the third fraction is less than the third decimal. This immediately tells us that the sum of decimals is greater, column B and answer **B**. Of course, to make this connection, it helps to recognize the decimal values of those fractions (a worthy study).

If doing the problem directly, I recommend adding the fractions, then adding the decimals and making one conversion at the end, rather than making three separate conversions.

5. We wish to determine the total number of marbles we receive. Again, a conversion is not necessary, we simply have to determine how many marbles each person gives us and sum up. Timothy gives us a fraction of his 100 marbles:

$$\frac{1}{50} \times 100 = 2 \tag{18}$$

2 marbles. How nice. Now Ursula and Victoria both give us the same amount (how convenient), 8%. After converting to a fraction, the conversion is similar:

$$\frac{8}{100} \times 100 = 8 \tag{19}$$

So, 8 marbles from each of them. This gives us $2+8+8 = 18$ marbles, answer **B**.

6. Finally, we wish to compare two values: 19% of x, and $\frac{0.5}{2.43} \times y$. All they tell us is that x and y are positive numbers. Two different positive numbers should be a huge red flag. The answer is almost certainly going to be D (not enough information). Let's check with some simple numbers to be sure:

Let $x = 100$. Then 19% of $x = 19$. Simple. Now, let $y = 2.43$, then:

$$\frac{0.5}{2.43} \times 2.43 = 0.5 \tag{20}$$

In this case, column A is greater. Let's find a y so that column B is greater. Instead of $y = 2.43$, try $y = 243$:

$$\frac{0.5}{2.43} \times 243 = \frac{0.5}{2.43} \times 2.43 \times 100 \tag{21}$$

$$= 0.5 \times 100 \tag{22}$$

$$= 50 \tag{23}$$

That settles it, the relationship cannot be determined from the information given, answer **D**.