Abstract Bracketology 1

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1 The One-Game Bracket

The least complicated analysis we can perform is on a bracket involving just a simple game. There is no room for complex strategy here, as we only need to consider the probability of each team winning and the probability of each person picking a given team. We begin by fixing a set of standard bracketology rules and some notation.

Rules of the One-Game Bracket

1. Each player places an equal bet to play (for simplicity we'll make the value of this bet equal to 1).
2. Each player selects one of the two teams to win the game. A correct choice earns 1 point, an incorrect choice earns 0 points.
3. If a player has a score strictly greater than all other players' scores, they win the entire pot.
4. If a subset of players have scores greater than or equal to all other players' scores, they split the pot evenly.

Now, let us denote the two teams A and B, and the probability of each team winning the game by p_A and p_B. We label each player (excluding ourself) by an integer $i \in \{1, 2, ..., n\}$, and denote the probability of a player selecting a particular team by c_j^i, where $i \in \{1, 2, ..., n\}$ and $j \in \{A, B\}$.

From the values c_j^i, we can calculate the expected number of (other) players selecting each team, which we denote N_A and N_B. Assuming each player chooses randomly based on the probabilities c_j^i, we have;

$$N_j = \sum_{i=1}^{n} c_j^i \tag{1}$$

These values will be important in determining the expectation value of any choice that we make.

Now, two things provide interesting dynamics here; the fact that ties split the pot and that this means our own decision changes the expectation value. Given how common ties are in a one-game bracket, we'll see that this generates some interesting phenomena, such as games which permit no choice of positive expectation value. We expect that as more games are introduced, and the number of ties decreases, the bracketology dynamics should change significantly.

Now the expectation value, in points, of choosing a particular team j is simply given by:

$$EV_{points}(j) = p_j \tag{2}$$

However, the bracketology rules are discrete in nature, so we need to interpret this information in a discrete way. Denote $P(x, j)$ to be the probability of scoring x points when choosing team j. We can then say;

$$P(0, j) = 1 - p_j \tag{3}$$

$$P(1, j) = p_j \tag{4}$$

If we were playing this game "in a bubble", that is, simply trying to maximize points rather than competing against other players, the optimal strategy would be obvious: simply choose the team with the greater value of p_j. However, the fact that other people are playing, presumably with access to the same information, complicates matters. The possibility of ties is also a major factor, especially in small tournaments. It plays an extreme role in the one-game bracket. Let us consider the expectation value, this time in dollars, of choosing a particular team j. There are three possibilities; that we will score 1 point, which will be (or tie) the highest score, that we will score 0 points, which will not be the highest score, and that we will score 0 points and tie for the highest score (this implies that everyone chooses the losing team). Now, we make the assumption that both N_A and N_B are greater than zero, eliminating the possibility of all players scoring zero. This yields:

$$EV(j) = p_j \frac{N_A + N_B}{N_j} \tag{5}$$

But wait. This doesn't take into account the fact that our play changes the percentage of players picking a particular team. Thus, the true expectation should be;

$$EV(j) = p_j \frac{N_A + N_B + 1}{N_j + 1} \tag{6}$$

Now, of course, our goal is to pick the the option with the greatest expectation. Before we decide to play, however, let's see if our expectation is greater than 1, taking team A as our example choice;

$$EV(A) \geq 1 \tag{7}$$

$$p_A \frac{N_A + N_B + 1}{N_A + 1} \geq 1 \tag{8}$$

$$\frac{N_A + N_B + 1}{N_A + 1} \geq \frac{1}{p_A} \tag{9}$$

$$\frac{N_A + 1}{N_A + N_B + 1} \leq p_A \tag{10}$$

Similarly, to justify playing the choice of team B, we should have that;

$$\frac{N_B + 1}{N_A + N_B + 1} \leq p_B \tag{11}$$